Quiz 15

October 28, 2016

Show all work and circle your final answer.

1. Check that each of the following functions satisfy the conditions of the Mean Value Theorem on the interval $[-1, 2]$. If it does, find all numbers c that satisfy Mean Value Theorem. If not, explain why it doesn't satisfy the conditions.

   (a) $f(x) = x^{3/4}$

   $f'(x) = \frac{3}{4}x^{-1/4} = \frac{3}{4\sqrt[4]{x}}$ OR $f(-1) = (-1)^{3/4} = \sqrt[4]{(-1)^3}$ DNE,

   $\boxed{f'(0) \text{ is undefined}}$ So $\boxed{\text{not continuous}}$ on $[-1,2]$

   So not diff. on $(-1,2)$

   (b) $f(x) = x^4 - 3x^2 + 1$ polynomials are continuous and differentiable everywhere. ✓

   $f'(x) = 4x^3 - 6x$

   $\frac{f(2)-f(-1)}{2-(-1)} = \frac{16-12+1-(1-3+1)}{3} = 2$

   Set $4x^3 - 6x = 2$

   $2(2x^3 - 3x - 1) = 0$

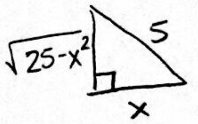
   -1 is a root, so $x+1$ is a factor:

   $2(x+1)(2x^2-2x-1) = 0$

   $\boxed{x=-1}$, $x = \frac{2 \pm \sqrt{4-4(2)(-1)}}{2(2)}$

   $= \boxed{\frac{1 \pm \sqrt{3}}{2}}$

2. A 5 foot ladder standing on level ground leans against a vertical wall. The bottom of the ladder is pulled away from the wall at 2 ft/sec. How fast is the AREA under the ladder changing when the top of the ladder is 4 feet above the ground?



$A = \frac{1}{2}x\sqrt{25-x^2}$

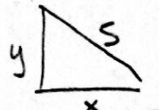
$\frac{dA}{dt} = \frac{1}{2}\sqrt{25-x^2}\frac{dx}{dt} + \frac{1}{2}x \cdot \frac{1}{2\sqrt{25-x^2}}(-x^2)\cdot\frac{dx}{dt}$

When $\sqrt{25-x^2} = 4$, $\frac{dx}{dt} = 2$ and $x = 3$

$\frac{dA}{dt} = \frac{1}{2}(4)(2) + \frac{1}{2}(3)\cdot\frac{1}{8}(-9)(2)$

$= 4 - \frac{9}{4} = \boxed{\frac{7}{4}}$

OR:



$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}\frac{dx}{dt}y = \boxed{\frac{7}{4}}$

when $y = 4$, $x = 3$, and

$x^2 + y^2 = 25$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(3)(2) + 2(4)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{3}{2}$

plug in $x, y, \frac{dx}{dt}, \frac{dy}{dt}$